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QUANTUM MECHANICS OF BARYOGENESIS

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Abstract

The cosmological baryon asymmetry can be explained as remnant of heavy Majorana neutrino decays in the early universe. We study this out-of-equilibrium process by means of Kadanoff-Baym equations which are solved in a perturbative expansion. To leading order the problem is reduced to solving a set of Boltzmann equations for distribution functions.

The generation of the cosmological matter-antimatter asymmetry in an expanding universe requires baryon number violation, C and CP violation, and a deviation from thermal equilibrium [1]. The classic mechanism which realizes these conditions is the decay of weakly interacting massive particles in a thermal bath [2]. Particularly successful is the leptogenesis scenario where the decaying particles are heavy Majorana neutrinos [3]. The resulting baryon asymmetry is entirely determined by neutrino properties. The observed order of magnitude can be naturally explained without any fine tuning of parameters and in accord with present experimental indications for neutrino masses [4].

The generation of a baryon asymmetry is an out-of-equilibrium process which is generally treated by means of Boltzmann equations. A thorough discription of the basic ideas can be found in [5]. Some subtleties have recently been discussed in [6]. A shortcoming of this approach is that the Boltzmann equations are classical equations for the time evolution of phase space distribution functions. On the contrary, the involved collision terms are S-matrix elements which involve quantum interferences of different amplitudes in a crucial manner. Clearly, a full quantum mechanical treatment is highly desirable. It is also required in order to justify the use of Boltzmann equations and to determine the size of corrections.

All information about the time evolution of a system is contained in the time dependence of its Green functions [7,8], which can be determined by means of Dyson-Schwinger equations. Originally these techniques were developed for non-relativistic many-body problems. More recently, they have also been applied to transport phenomena in nuclear matter [9], the electroweak plasma [10,11] and the QCD plasma [12]. Alternatively, one may study the time evolution of density matrices [13,14]. In the following we shall investigate non-equilibrium Green functions which are relevant for leptogenesis. We shall construct a perturbative solution of the corresponding Kadanoff-Baym equations which, to leading order, turn out to be equivalent to a set of Boltzmann equations. Higher-order corrections can then be systematically evaluated.

Consider now the standard model with three additional right-handed neutrinos whose interactions are described by the lagrangian,

$$\mathcal{L} = \bar{l}_L \widetilde{\phi} \lambda^* \nu_R - \frac{1}{2} \overline{\nu}^c_R M \nu_R + h.c. \tag{1}$$

Here l_L and ϕ denote lepton and Higgs doublets, respectively. We shall restrict our discussion to the case of hierarchical Majorana neutrino masses, $M_1 \ll M_2, M_3$. The baryon asymmetry will then be determined by the CP violating decays of the lightest Majorana neutrino $N_1 = \nu_{R1} + \nu^c_{R1} \equiv N$,

$$\Gamma(N \to l\phi) = \frac{1}{2}(1+\epsilon)\Gamma\ , \quad \Gamma(N \to \bar{l}\bar{\phi}) = \frac{1}{2}(1-\epsilon)\Gamma\ . \tag{2}$$

Here Γ is the total decay width and the parameter $\epsilon \ll 1$ measures the amount of CP violation. The generation of the baryon asymmetry takes place at a temperature

$T \sim M_1 \equiv M \ll M_2, M_3$. It is therefore convenient to describe the system by an effective lagrangian where the two heavier neutrinos have been integrated out,

$$\begin{aligned}\mathcal{L} &= \bar{l}_{Li}\widetilde{\phi}\lambda^*_{i1}N + N^T\lambda_{i1}Cl_{Li}\phi - \frac{1}{2}MN^TCN \\ &\quad + \frac{1}{2}\eta_{ij}l^T_{Li}\phi\ C\ l_{Lj}\phi + \frac{1}{2}\eta^*_{ij}\bar{l}_{Li}\widetilde{\phi}\ C\ \bar{l}^T_{Lj}\widetilde{\phi}\ ,\end{aligned} \tag{3}$$

with

$$\eta_{ij} = \sum_{k=2}^{3}\lambda_{ik}\frac{1}{M_k}\lambda^T_{kj}\ . \tag{4}$$

For leptogenesis one has to consider the phase space distributions for heavy neutrinos (f_N), leptons (f_l), anti-leptons ($f_{\bar{l}}$), Higgs (f_ϕ) and anti-Higgs bosons ($f_{\bar{\phi}}$). The generation of the lepton asymmetry is a process close to equilibrium. Hence one can linearize the Boltzmann equations in the deviations from the equilibrium distributions. Due to the interactions in (3) one has $\delta f_l = -\delta f_{\bar{l}} = \delta f_\phi = -\delta f_{\bar{\phi}}$. The Boltzmann equation for the Majorana neutrino reads

$$\begin{aligned}&g_N\frac{\partial}{\partial t}\delta f_N(t,p) = \\ &-g_N\frac{\partial}{\partial t}f_N(p) - \frac{1}{2E}\int d\Phi_{\bar{1}\bar{2}}(p)\delta f_N(t,p)\left(|\mathcal{M}(N\to l\phi)|^2 + |\mathcal{M}(N\to \bar{l}\bar{\phi})|^2\right).\end{aligned} \tag{5}$$

For the lepton doublets one obtains

$$\begin{aligned}&2g_l\frac{\partial}{\partial t}\delta f_l(t,k) = \\ &\frac{1}{2k}\int d\Phi_{\bar{1}2}(k)\ \epsilon\ \delta f_N(t,p_1)\left(|\mathcal{M}(N\to l\phi)|^2 + |\mathcal{M}(N\to \bar{l}\bar{\phi})|^2\right) \\ &-\frac{1}{2k}\int d\Phi_{1\bar{2}}(k)\left(\delta f_l(t,k)f_\phi(p_1) + f_l(k)\delta f_\phi(t,p_1)\right) \\ &\qquad\qquad \times\left(|\mathcal{M}(l\phi\to N)|^2 + |\mathcal{M}(\bar{l}\bar{\phi}\to N)|^2\right) \\ &-\frac{1}{2k}\int d\Phi_{1\bar{2}\bar{3}}(k)\left(\delta f_l(t,k)f_\phi(p_1) + f_l(k)\delta f_\phi(t,p_1)\right) \\ &\qquad\qquad \times\left(|\mathcal{M}(l\phi\to \bar{l}\bar{\phi})|^2 + |\mathcal{M}(\bar{l}\bar{\phi}\to l\phi)|^2\right) \\ &-\frac{1}{2k}\int d\Phi_{\bar{1}\bar{2}3}(k)\left(\delta f_l(t,p_1)f_\phi(p_2) + f_l(p_1)\delta f_\phi(t,p_2)\right) \\ &\qquad\qquad \times\left(|\mathcal{M}(l\phi\to \bar{l}\bar{\phi})|^2 + |\mathcal{M}(\bar{l}\bar{\phi}\to l\phi)|^2\right) \\ &-\frac{1}{4k}\int d\Phi_{1\bar{2}\bar{3}}(k)\left(\delta f_l(t,k)f_l(p_1) + f_l(k)\delta f_l(t,p_1)\right) \\ &\qquad\qquad \times\left(|\mathcal{M}(ll\to \bar{\phi}\bar{\phi})|^2 + |\mathcal{M}(\bar{l}\,\bar{l}\to \phi\phi)|^2\right) \\ &-\frac{1}{2k}\int d\Phi_{\bar{1}\bar{2}3}(k)\delta f_\phi(t,p_1)f_\phi(p_2)\left(|\mathcal{M}(\phi\phi\to \bar{l}\,\bar{l})|^2 + |\mathcal{M}(\bar{\phi}\bar{\phi}\to ll)|^2\right)\ .\end{aligned} \tag{6}$$

Here

$$d\Phi_{1\dots\bar{n}\dots}(p) = \frac{d^3p_1}{(2\pi)^3 2E_1} \cdots \frac{d^3p_{\bar{n}}}{(2\pi)^3 2E_{\bar{n}}} \cdots (2\pi)^4 \delta^4(p + p_1 + \dots - p_{\bar{n}} - \dots) \; , \qquad (7)$$

$$f_i(p) = \exp\left(-\beta E_i(p)\right) \; , \qquad (8)$$

denote phase space integrations and distribution functions, respectively. The temperature $T = 1/\beta$, and $\mathcal{M}(\dots)$ is the matrix element of the indicated process. $g_N = 2$ and $g_l = g_{\bar{l}} = 6$ are the number of 'internal' degrees of freedom for the Majorana neutrino and the lepton doublets for three generations, respectively. For simplicity we have assumed small number densities so that we can use Boltzmann distribution functions for bosons and fermions and also neglect distribution functions for particles in the final state. The effect of the Hubble expansion is included by introducing the 'covariant' derivative $\partial/\partial t \to \partial/\partial t - Hp\partial/\partial p$. Integration over momenta then yields the more familiar form of the Boltzmann equations for the number densities. Eq. (5) describes the decay of the heavy Majorana neutrinos. Note, that also the equilibrium distributions are time dependent since the temperature varies with time. For massless particles the distribution functions are constant with respect to the 'covariant' time derivative. The first term in eq. (6) drives the generation of a lepton asymmetry; the remaining terms tend to wash out an existing asymmetry. Eqs. (5) and (6) determine δf_N and δf_l as function of time. We have only kept the interactions given by the lagrangian (3). A complete discussion can be found in [15].

Green functions near thermal equilibrium

The time evolution of an arbitrary multi-particle lepton-Higgs system can be studied by means of the Green functions of lepton and Higgs fields. For the heavy Majorana neutrino one has

$$iG_{\alpha\beta}(x_1, x_2) = \mathrm{Tr}\left(\rho T N_\alpha(x_1) N_\beta(x_2)\right) \; , \qquad (9)$$

where T denotes the time ordering, ρ is the density matrix of the system, the trace extends over all states, and the time coordinates t_1 and t_2 lie on an appropriately chosen contour C in the complex plane [16]. $G(x_1, x_2)$ can be written as a sum of two parts,

$$G(x_1, x_2) = \Theta(t_1 - t_2) G^>(x_1, x_2) + \Theta(t_2 - t_1) G^<(x_1, x_2) \; , \qquad (10)$$

where

$$iG^>(x_1, x_2)_{\alpha\beta} = \mathrm{Tr}\left(\rho N_\alpha(x_1) N_\beta(x_2)\right) , \; iG^<(x_1, x_2)_{\alpha\beta} = -\mathrm{Tr}\left(\rho N_\beta(x_2) N_\alpha(x_1)\right) \; . \quad (11)$$

The 'time ordering' in eq. (10) is along the contour C.

For a system in thermal equilibrium at a temperature $T = 1/\beta$ the density matrix is $\rho = \exp(-\beta H)$, where H is the Hamilton operator. In this case the Green function

only depends on the difference of coordinates and it is convenient to introduce the Fourier transform,

$$G(p) = \int d^4x e^{ipx} G(x) \ . \tag{12}$$

The contour C can be chosen as a sum of two branches, $C = C_1 \cup C_2$, which lie above and below the real axis. The time coordinates are real and associated with one of the two branches. Correspondingly, the Green function becomes a 2×2 matrix,

$$G(p) = \begin{pmatrix} G^{11}(p) & G^{12}(p) \\ G^{21}(p) & G^{22}(p) \end{pmatrix} \ . \tag{13}$$

The off-diagonal terms are given by

$$G^{12}(p) = G^{<}(p) \ , \quad G^{21}(p) = G^{>}(p) \ . \tag{14}$$

The diagonal terms of the matrix (13) are the familiar causal and anti-causal Green functions. The functions $G^{>}(p)$ and $G^{<}(p)$ satisfy the KMS-condition,

$$G^{<}(p) = -e^{-\beta p_0} G^{>}(p) \ , \tag{15}$$

and the free Green functions are explicitly given by

$$iG^{>}(p) \ = \ \left(\Theta(p_0) - \Theta(p_0) f_N(E) - \Theta(-p_0) f_{\bar{N}}(E)\right) \rho_N(p) \ , \tag{16}$$

$$iG^{<}(p) \ = \ \left(\Theta(-p_0) - \Theta(p_0) f_N(E) - \Theta(-p_0) f_{\bar{N}}(E)\right) \rho_N(p) \ , \tag{17}$$

with the spectral density

$$\rho_N(p) = 2\pi(\not{p} + M) C^{-1} \delta(p^2 - M^2) \ , \tag{18}$$

and the Fermi-Dirac distribution functions

$$f_N(E) = f_{\bar{N}}(E) = \frac{1}{e^{\beta E} + 1} \ , \quad E = \sqrt{M^2 + p^2} \ . \tag{19}$$

Since $N(x)$ is a Majorana field one has $f_N = f_{\bar{N}}$, and in the spectral density (18) the charge conjugation matrix C occurs. In the following we shall also need the retarded and advanced Green functions,

$$G^{\pm}(x) = \pm\Theta(\pm x^0) \left(G^{>}(x) - G^{<}(x)\right) \ , \tag{20}$$

which can be written as sum of an on-shell and an off-shell contribution,

$$G^{\pm}(p) = \pm\frac{1}{2}\left(G^{>}(p) - G^{<}(p)\right) + \frac{1}{2\pi i}\mathcal{P}\int d\omega' \frac{G^{>}(x,\omega',\vec{p}) - G^{<}(x,\omega',\vec{p})}{\omega - \omega'} \ . \tag{21}$$

The Green functions for the lepton doublets and for the Higgs doublet,

$$iS(x_1,x_2)_{\alpha\beta}\delta^a_b = \mathrm{Tr}\left(\rho T l^a_\alpha(x_1) \bar{l}_{b\beta}(x_2)\right) , \ i\Delta(x_1,x_2)\delta^a_b = \mathrm{Tr}\left(\rho T \phi^a(x_1)\phi^*_b(x_2)\right) , \tag{22}$$

have the same structure as $G(x_1, x_2)$. The corresponding equations for $S(p)$ are obtained from eqs. (10)-(19) by replacing the spectral density $\rho_N(p)$ by

$$\rho_l(p) = 2\pi P_L \not{p}\,\delta(p^2 - M^2)\ , \quad P_L = \frac{1-\gamma_5}{2}\ , \tag{23}$$

and the distribution functions $f_N(E)$ and $f_{\bar{N}}(E)$ by

$$f_l(E, \mu_l) = f_{\bar{l}}(E, -\mu_l) = \frac{1}{e^{\beta(E-\mu_l)} + 1}\ , \quad E = |\vec{p}|\ , \tag{24}$$

where μ_l is the lepton chemical potential. For the Higgs field one has

$$\rho_\phi(p) = 2\pi\delta(p^2 - M^2)\ , \tag{25}$$

$$f_\phi(E, \mu_\phi) = f_{\bar{\phi}}(E, -\mu_\phi) = \frac{1}{e^{\beta(E-\mu_\phi)} - 1}\ , \quad E = |\vec{p}|\ . \tag{26}$$

We are considering a process close to equilibrium. This suggests that the corresponding deviations of the Green functions may be obtained from the equilibrium Green functions by a small change of the distribution functions,

$$i\delta G(x, p) = -\delta f_N(x, p)\rho_N(p)\ , \tag{27}$$

$$i\delta S(x, k) = -\epsilon(k_0)\delta f_l(x, k)\rho_l(k)\ , \quad i\delta\Delta(x, q) = -\epsilon(q_0)\delta f_\phi(x, q)\rho_\phi(q)\ . \tag{28}$$

Here we have used that due to the interactions given in (3) $\delta f_l = -\delta f_{\bar{l}} = \delta f_\phi = -\delta f_{\bar{\phi}}$.

Kadanoff-Baym equations

The Green functions for the heavy neutrino and the leptons satisfy Dyson-Schwinger equations,

$$\begin{aligned} C(i\not{\partial}_1 - M)G(x_1, x_2) &= \delta(x_1 - x_2) + \int_C d^4x_3 \Sigma(x_1, x_3)G(x_3, x_2)\ , &(29)\\ i\not{\partial}_1 S(x_1, x_2) &= \delta(x_1 - x_2) + \int_C d^4x_3 \Pi(x_1, x_3)S(x_3, x_2)\ , &(30)\end{aligned}$$

where Σ and Π are the corresponding self energies and the time integration is carried out along the contour C. Eqs. (29) and (30) can be turned into matrix equations with real time integration in the usual manner. For the off-diagonal elements $G^>$ and $S^>$ one then obtains

$$\begin{aligned} C(i\not{\partial}_1 - M)G^>(x_1, x_2) &= \int d^4x_3 \Big(\Sigma^>(x_1, x_3)G^-(x_3, x_2) \\ &\qquad +\Sigma^+(x_1, x_3)G^>(x_3, x_2)\Big)\ , &(31)\\ i\not{\partial}_1 S^>(x_1, x_2) &= \int d^4x_3 \Big(\Pi^>(x_1, x_3)S^-(x_3, x_2) \\ &\qquad +\Pi^+(x_1, x_3)S^>(x_3, x_2)\Big)\ . &(32)\end{aligned}$$

Here $\Sigma^>$ and $\Pi^>$ are off-diagonal matrix elements of Σ and Π, which are defined analogous to $G^>$. The equations for $G^<$ and $S^<$ can be obtained from eqs. (31) and (32) by replacing the superscripts '$>$' by '$<$'. Equations of the type (31), (32) have first been obtained by Kadanoff and Baym for non-relativistic many-body systems [7].

For processes where the overall time evolution is slow compared to relative motions the Kadanoff-Baym equations can be solved in a derivative expansion. One considers the Wigner transform for $G(x_1, x_2)$,

$$G(x,p) = \int d^4y\ e^{ipy}\ G\left(x + \frac{y}{2}, x - \frac{y}{2}\right)\ , \tag{33}$$

and $S(x,p)$, $\Delta(x,p)$, respectively. For the Wigner transform of a convolution one has in general,

$$\begin{aligned}&\int d^4y\ e^{ipy} \int d^4x_2\ A(x_1,x_2)B(x_2,x_3) \qquad (34)\\ &= A(x,p)B(x,p) - \frac{i}{2}\left(\frac{\partial}{\partial x}A(x,p)\frac{\partial}{\partial p}B(x,p) - \frac{\partial}{\partial x}B(x,p)\frac{\partial}{\partial p}A(x,p)\right) + \dots\ ,\end{aligned}$$

where $x = (x_1 + x_3)/2$ and $y = (x_1 - x_3)/2$.

Using the derivative expansion (34) the Kadanoff-Baym equations (31) and (32) become local in the space-time coordinate x. Keeping to zeroth order only the on-shell part of retarded and advanced Green functions and self-energies, which are given by expressions analogous to eq. (21), one obtains the equations

$$\begin{aligned}C(\frac{i}{2}\partial\!\!\!/ + p\!\!\!/ - M)G^>(x,p) &= C(\frac{i}{2}\partial\!\!\!/ + p\!\!\!/ - M)G^<(x,p)\\ &= \frac{1}{2}\left(\Sigma^>(x,p)G^<(x,p) - \Sigma^<(x,p)G^>(x,p)\right)\ , \qquad (35)\\ (\frac{i}{2}\partial\!\!\!/ + k\!\!\!/)S^>(x,k) &= (\frac{i}{2}\partial\!\!\!/ + k\!\!\!/)S^<(x,k)\\ &= \frac{1}{2}\left(\Pi^>(x,k)S^<(x,k) - \Pi^<(x,k)S^>(x,k)\right)\ . \qquad (36)\end{aligned}$$

Solutions of these equations yield the first terms for the non-equilibrium Green functions $G^>(x,p)...S^<(x,k)$ in an expansion involving off-shell effects and space-time variations, which include 'memory effects'. As an example for the type of corrections we list the first derivative term on the right-hand side of eq. (35),

$$\begin{aligned}\Delta_\partial = &-\frac{i}{4}\left(\frac{\partial}{\partial x}\Sigma^>(x,p)\frac{\partial}{\partial p}G^<(x,p) - \frac{\partial}{\partial p}\Sigma^>(x,p)\frac{\partial}{\partial x}G^<(x,p)\right.\\ &\left.-\frac{\partial}{\partial x}\Sigma^<(x,p)\frac{\partial}{\partial p}G^>(x,p) + \frac{\partial}{\partial p}\Sigma^<(x,p)\frac{\partial}{\partial x}G^>(x,p)\right)\ . \qquad (37)\end{aligned}$$

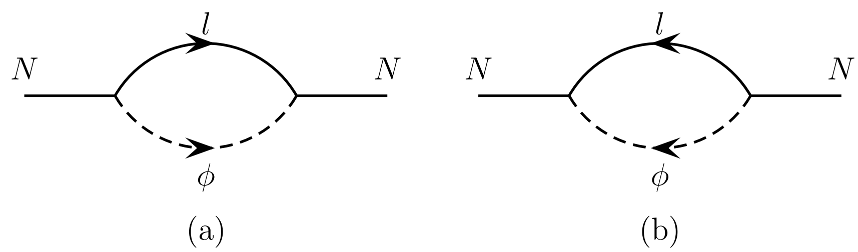

Figure 1: *One-loop self energies for the Majorana neutrino.*

Solutions of the Kadanoff-Baym equations can be studied once the self-energies Σ and Π are known. For weak coupling these can be determined in perturbation theory.

Self-energies for lepton fields

The one-loop contributions to the self-energy of the Majorana neutrino are shown in fig. (1). For vanishing chemical potential fig. (1a), for instance, yields the result

$$-i\Sigma_{eq}^{(a)>}(p) = 2(\lambda^\dagger\lambda)_{11}\int \frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}(2\pi)^4\delta(p-p_1-p_2)CS^>(p_1)\Delta^>(p_2)\ . \quad (38)$$

In the following we shall only consider the case $M \gg T$, where the heavy neutrinos are non-relativistic. In this case the number density is small, $f_N(E) \ll 1$, and one finds for the difference of the self-energies ($p_0 > 0$),

$$\begin{aligned} -i(\Sigma_{eq}^>(p)-\Sigma_{eq}^<(p)) &= -2(\lambda^\dagger\lambda)_{11}\int d\Phi_{\bar{1}\bar{2}}(p)C\not{p}_1 \\ &\quad \times((1-f_l(p_1))(1+f_\phi(p_2))+f_l(p_1)f_\phi(p_2)) \quad (39) \\ &\simeq -\Gamma C\frac{\not{p}}{M}\ . \quad (40) \end{aligned}$$

Here Γ is the vacuum decay rate of the Majorana neutrino.

The one- and two-loop contributions to the lepton self-energy are shown in fig. (2a)-(2d). In the following we only list the terms which are needed for the solution of the Kadanoff-Baym equations to leading order.

Particularly interesting are the terms fig. (2b) which drive the generation of an asymmetry. After some algebra one finds ($k_0 > 0$),

$$-i\left(\delta\Pi^{(b)<}(k)+\delta\Pi^{(b)>}(-k)\right) = \frac{3}{4\pi}\mathrm{Im}(\lambda^\dagger\eta\lambda^*)_{11}M\int d\Phi_{1\bar{2}}(k)\not{p}_2P_L\delta f_N(t,p_2)\ . \quad (41)$$

Here we have only given the deviation from the equilibrium self-energy which is obtained by using for the Majorana neutrino propagator the deviation from the equilibrium propagator $\delta G \propto \delta f_N$. Furthermore, we have again considered the case of

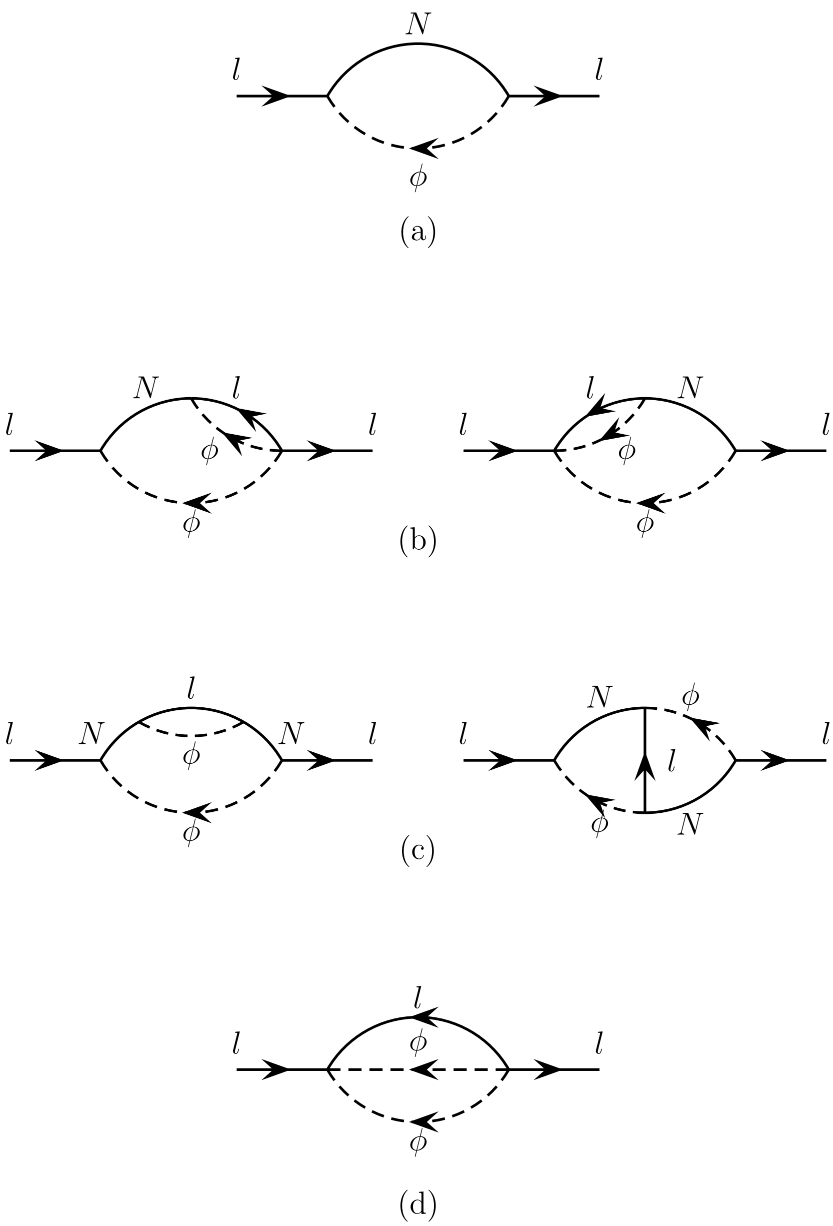

Figure 2: *One- and two-loop self energies for the lepton doublet.*

small densities. The combination of Yukawa coulings is precisely the one occuring in the CP asymmetry of the Majorana neutrino decay. Since the heavy neutrinos N_2 and N_3 have been integrated out, the contributions fig. (2b) involve both, self-energy and vertex corrections [17,18,19] which, up to a numerical factor, are identical in this limit.

All graphs in fig. (2) contain washout processes. In the case of small densities one obtains from fig. (2a) ($k_0 > 0$),

$$-i\left(\Pi_{eq}^{(a)>}(k) - \Pi_{eq}^{(a)<}(k)\right) = -2(\lambda^\dagger\lambda)_{11}\int d\Phi_{1\bar{2}}(k)\not{p}_2 P_L(f_\phi(p_1) + f_N(p_2)) \ . \tag{42}$$

For $T \ll M$ the dominant contribution to the washout processes is due to fig. (2c) where the Majorana neutrino propagator can be replaced by a local interaction. For small densities one finds ($k_0 > 0$),

$$-i\left(\Pi_{eq}^{(c)>}(k) - \Pi_{eq}^{(c)<}(k)\right) = -6\frac{(\lambda^\dagger\lambda)_{11}^2}{M^2}\int d\Phi_{1\bar{2}\bar{3}}(k)\left(\not{p}_1 f_l(p_1) + 2\not{p}_2 f_\phi(p_1)\right)P_L \ . \tag{43}$$

The complete expressions for the self-energies will be given elsewhere.

Kinetic equations

Given the lepton self-energies we can now look for solutions of the Kadanoff-Baym equations (35) and (36). A straightforward calculation shows that the right-hand side of these equations vanishes for equilibrium Green functions and self-energies. Since baryogenesis is a process close to thermal equilibrium we can search for solutions which are linear in the deviations,

$$\delta G(t,p) = G^>(t,p) - G_{eq}^>(p) = G^<(t,p) - G_{eq}^<(p) \ , \tag{44}$$
$$\delta S(t,p) = S^>(t,p) - S_{eq}^>(p) = S^<(t,p) - S_{eq}^<(p) \ . \tag{45}$$

One then obtains for the perturbations $\delta G(t,p)$ and $\delta S(t,p)$,

$$iC\gamma^0\frac{\partial}{\partial t}\delta G(t,p) = iC\gamma^0\frac{\partial}{\partial t}G_{eq}^>(p) + (\Sigma_{eq}^>(p) - \Sigma_{eq}^<(p))\delta G(t,p) \ , \tag{46}$$

$$\begin{aligned} i\gamma^0\frac{\partial}{\partial t}\delta S(t,k) &= (\Pi_{eq}^>(k) - \Pi_{eq}^<(k))\delta S(t,k) \\ &\quad + \delta\Pi^>(t,k)S_{eq}^<(k) - \delta\Pi^<(t,k)S_{eq}^>(k) \ . \end{aligned} \tag{47}$$

The Green functions depend on time explicitly, as well as implicitly through the time-dependence of the temperature. Once the 'covariant' time derivative is used, the later vanishes for equilibrium Green functions of massless fields. This is not the case for massive fields. Hence, the first term on the right-hand side of (46) drives the deviation from thermal equilibrium.

We can now insert the perturbative expressions for the self-energies into eqs. (46), (47) and check whether the ansatz (27), (28) for δG and δS yields a solution. After some algebra one finds that this is indeed the case provided the distribution functions δf_N and δf_l satisfy the following ordinary differential equations,

$$
\begin{aligned}
E\frac{\partial}{\partial t}\delta f_N(t,p) &= -E\frac{\partial}{\partial t}f_N(p) - 2(\lambda^\dagger\lambda)_{11}\int d\Phi_{\bar{1}\bar{2}}(p)\delta f_N(t,p)p\cdot p_1\ , \qquad (48)\\
g_l k\frac{\partial}{\partial t}\delta f_l(t,k) &= \frac{3}{8\pi}\mathrm{Im}(\lambda^\dagger\eta\lambda^*)_{11}M\int d\Phi_{\bar{1}2}(k)\delta f_N(t,p_1)k\cdot p_1\\
&\quad -2(\lambda^\dagger\lambda)_{11}\int d\Phi_{1\bar{2}}(k)\left(\delta f_l(t,k)f_\phi(p_1)+f_l(k)\delta f_\phi(t,p_1)\right)k\cdot p_2\\
&\quad -6\frac{(\lambda^\dagger\lambda)_{11}^2}{M^2}\int d\Phi_{1\bar{2}\bar{3}}(k)\Big(2(\delta f_l(t,k)f_\phi(p_1)+f_l(k)\delta f_\phi(t,p_1)\\
&\qquad +\delta f_l(t,p_2)f_\phi(p_3)+f_l(p_2)\delta f_\phi(t,p_3))k\cdot p_2\\
&\qquad +(\delta f_l(t,k)f_l(p_1)+f_l(k)\delta f_l(t,p_1)\\
&\qquad\quad +2\delta f_\phi(t,p_2)f_\phi(p_3))k\cdot p_1\Big)\ . \qquad (49)
\end{aligned}
$$

Comparing these equations with the Boltzmann equations (5) and (6) one finds that the two sets of equations are identical to leading order in the coupling where matrix elements and CP asymmetry are given by

$$
\begin{aligned}
|\mathcal{M}(N(p)\to l(p_1)\phi(p_2))|^2 &= 4(\lambda^\dagger\lambda)_{11}\ p\cdot p_1\ , \qquad (50)\\
|\mathcal{M}(l(k)\phi(p_1)\to \bar{l}(p_2)\bar{\phi}(p_3))|^2 &= 24\frac{(\lambda^\dagger\lambda)_{11}^2}{M^2}\ k\cdot p_2\ , \qquad (51)
\end{aligned}
$$

$$
\epsilon = \frac{3}{16\pi}\frac{\mathrm{Im}(\lambda^\dagger\eta\lambda^*)_{11}}{(\lambda^\dagger\lambda)_{11}}M\ . \qquad (52)
$$

We conclude that for non-relativistic heavy neutrinos a solution of the Boltzmann equations generates a solution of the full Kadanoff-Baym equations to leading order in the expansion described above. For relativistic heavy neutrinos the matrix structure of the equations is more complicated and the time evolution of the different poles of the Majorana neutrino propagator are described by different equations.

Given a solution of the Kadanoff-Baym equations to leading order the various corrections can be systematically studied. Note, that the solutions of eqs. (48) and (49) are not of the form $\delta f_i(t,p) = h_i(t)f_i(p)$. Hence, the usual assumption of kinetic equilibrium does not appear to be justified. The size of 'derivative terms', which correspond to memory effects, and off-shell corrections can be determined by inserting the leading order solution into the various correction terms described above. Particularly interesting are relativistic corrections in the case that leptogenesis takes place at temperatures $T \sim M$.

The analysis of the Kadanoff-Baym equations for leptogenesis can be used to obtain constraints on the parameters M, $(\lambda^\dagger\lambda)_{11}$ and ϵ, which provides a quantitative relation between the cosmological baryon asymmetry and neutrino properties.

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